A GENERAL FRAMEWORK FOR STOCHASTIC INVESTIGATIONS OF MORTALITY AND INVESTMENT RISKS

P J LEE

ABSTRACT

The paper considers two of the principal risks to which life insurers and pension plans are subject (mortality and interest rate/investment performance risks) and sets out (taking as an example a portfolio of annuity business) a general framework for analyzing such risks and investigating strategies to manage them. The framework consists of submodels for the constituent parts of the problem (including data, mortality rates, the economic environment, investment strategy and the reserving behaviour adopted by management). A general framework is useful because it enables substitution of alternative submodels without disturbing the remainder of the overall model. Specimen calculation results are provided.

KEYWORDS

Stochastic asset models, stochastic mortality models, reserving models, framework, polymorphism, Wilkie model, Smith model, Cairns model.

CONTACT ADDRESS

P.J. Lee, InQA Ltd, 42 Chatsworth Road, Bournemouth, BH8 8SW, UK.
Tel: +44-1202-528840; Email: PatrickLee@inqa.com; Web: www.inqa.com.

1 INTRODUCTION

1.1 In the last five years in the United Kingdom, the risks of managing portfolios of annuities have attracted increasing attention. This is a result of many factors, including an ageing population, problems arising from guaranteed annuity rates in an environment where interest rates are now much lower than at the time of issue of such guarantees, and improvements in longevity rates significantly beyond those expected (see CMIR 1990 and 1999, also Willets, 1999).

1.2 In this as in many other areas, it is obvious that the world is changing, and managing such change is perhaps one the most significant challenges of today. The analysis and management of the stochastic economic and demographic risks faced by insurance companies, pension funds and others is a complex task. The availability of fast modern computers and appropriate software has a significant role to play in the management of this task, due to the volume of calculations that are involved.

1.3 However, there is significant work involved in creating the necessary software. Given the likelihood of future changes in the different methodologies and tools available to analyse the different parts of the problem (e.g. separate models for economic, mortality and reserving outcomes), it is important to maximise the re-usability of such software. The use of object-oriented computer programming techniques, and the design of a framework which will prove to be durable under a wide range of future changes, is therefore a significant contribution to the process of managing change.

1.4 The purpose in this paper is to outline the approach taken by the author's firm to solve such problems in practice, taking as an example a portfolio of annuity business, and to demonstrate some initial calculation results.

2 APPROACH ADOPTED

2.1 *Polymorphism and the use of a Framework*

2.1.1 The approach I have adopted follows that used for our Global Risk Manager (GRM) software, which is an object-oriented suite of asset liability management tools. GRM makes use of *polymorphism* and a proprietary overall *framework*. In systems analysis, a *framework* makes up a re-usable design for software for a specific problem domain. *Polymorphism* is an object-oriented concept which allows existing software components within a framework to be replaced by future components without any changes to other components.

2.2 *Examples of Polymorphism in this context*

2.2.1 For the particular problem domain under consideration, namely the management of annuity business, examples of the component replacements that might be required include:

- new stochastic asset models
- new stochastic mortality models
- new models of reserving behaviour.

A polymorphic design enables changes to be made in any of these areas without any changes to any other components, including higher level components which use the services of lower level components. An example of a higher level component would be a model of the whole portfolio of annuity business and its development over time. Examples of the services provided by lower level components include zero coupon prices in the case of a stochastic asset model, or the stochastic mortality experience of a group of annuitants in the case of a stochastic mortality model.

2.3 *Overview of the Framework*

2.3.1 An essential part of the framework is to break the problem of modelling the behaviour of the whole portfolio of annuity business into submodels for the constituent parts of the problem. These include:

- data storage
- stochastic mortality: mortality rates
- stochastic asset model: the economic environment
- investment strategy
- the reserving behaviour adopted by management

A well designed system needs to be robust with respect to the changes that will almost inevitably arise in future any of these areas. Figure 2.3 shows a simplified view of the framework (in particular, items such as investment strategy and expenses are not shown), including the areas of most likely future change. In the next section, we outline each of the principal submodels in turn.

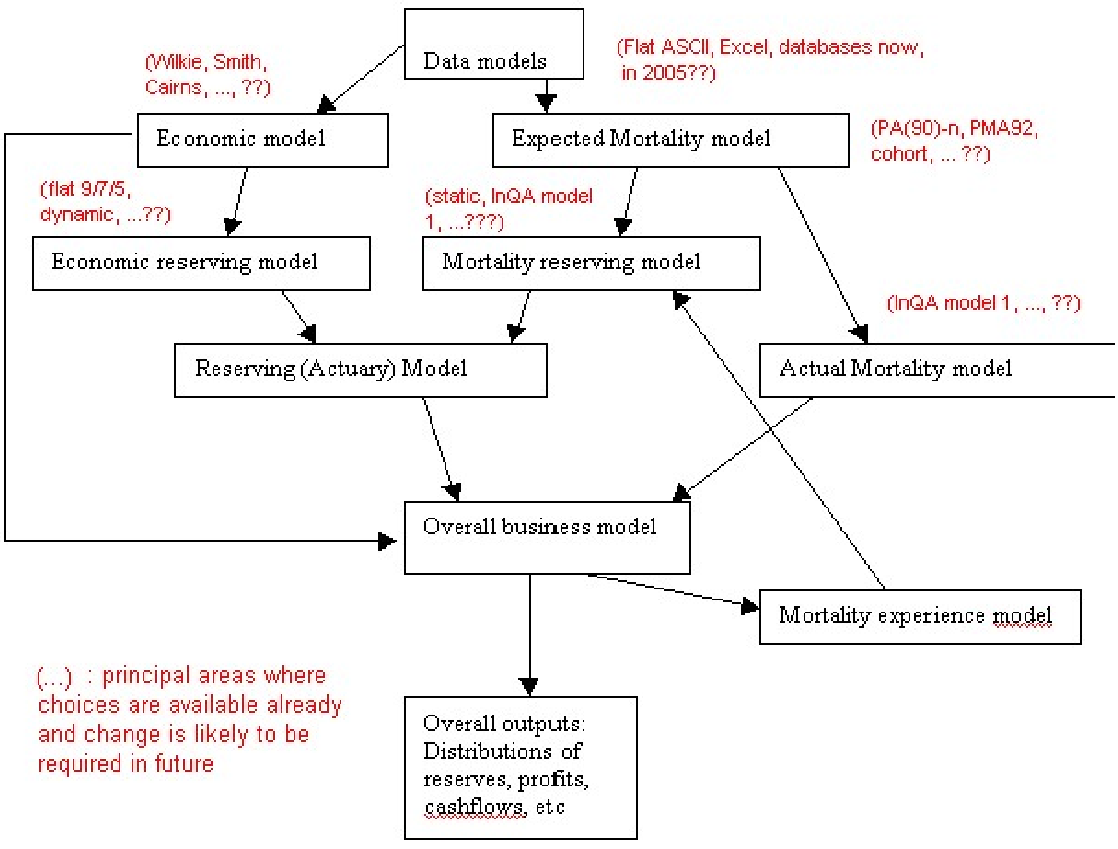

Figure 2.3: a simplified pictorial view of the framework

3 SUBMODELS USED

3.1 *Data storage*

3.1.1 A flexible system needs to be robust under changes to the location of the data which is stored. For example, higher level components within the system should be indifferent (in the sense of not requiring modification under changes) as to whether the necessary data is stored in a flat ASCII file, a Microsoft Excel workbook, or a relational database such as Oracle or Microsoft Access. Examples of the data fields which might be required include total and average pension amounts at each age attained, split into several pension types, each of which attracts different future pension increases. This assumes that data is grouped by age attained, and the use of average pensions enables exposure by amount to be approximated as total pension / average pension.

3.2 *Stochastic Mortality*

3.2.1 The purpose of the stochastic mortality submodel is to enable expected and actual (simulated) mortality rates to be calculated. The system should be robust under changes to the detailed formulae used to calculate such outputs.

3.2.2 Examples of different approaches which might be taken to calculate **expected** mortality rates include:

- the use of a fixed mortality table, such as PA(90), with a given age rating, but without any further allowance for future mortality improvements by calendar year and year of birth
- the use of the more recent tables published by the Continuous Mortality Investigation Bureau (CMIR 1990 and 1999), such as the 80 and 92 series of tables, which include explicit formulae for future mortality improvements (CMIR 1990 and 1999)
- the use of cohort based tables (Willets 1999).

3.2.3 The approach I have adopted to modelling **actual** (simulated) mortality rates is as follows, following suggestions by my colleague, David Wilkie. Given $q_{0x}(t)$ an expected probability of death during calendar year t for an individual with age attained x, I assume that the true probability of death, $q_{1x}(t)$ is unobservable, but may be approximated as follows:

$q_{1x}(t) = q_{0x}(t) * \exp(Y(t))$ where

$Y(t) = X(t) + e_Y(t)$,

$X(t) = X(t-1) + e_X(t)$,

$e_Y(t)$ and $e_X(t)$ are i.i.d. $N(0, \sigma_Y^2)$ and $N(0, \sigma_X^2)$ respectively i.e. we assume that a Kalman filter-like process is applied to a true but unobservable random mortality component X(t) (which is a random walk series), to arrive at Y(t), the observable part of random mortality X(t).

3.2.4 For a given annuitant population, I then calculate **actual** stochastic mortality on the assumption that the actual number of deaths $q_{2x}(t)$ is distributed N (n q_{1x} (t) , n q_{1x} (t) (1- q_{1x} (t)), where n is the exposed to risk by amounts (taken in practice as total pension at age attained x / average pension at age attained x). I am effectively assuming that the distribution of actual deaths is binomially distributed and taking the normal approximation N(nq, npq) to the binomial for large n.

3.2.5 Suggestions as to alternative models for simulating actual mortality rates and outcomes will be gratefully received. For the avoidance of ambiguity in future discussions about alternative mortality models, I suggest that the model described in 3.2.3 and 3.2.4 be called InQA mortality model 1. (This coincides with the name of the programming component that implements this; the naming convention has the advantage that subsequent models can be readily added and distinguished one from another.)

3.3 *Stochastic Asset Models*

3.3.1 The purpose of the stochastic asset model is to provide economic variables which are necessary for:

- projecting future pension amounts (e.g. price inflation)
- calculating the reserves required to meet future liabilities (e.g. nominal and inflation linked zero coupon prices)
- calculating the return on the portfolio of assets held given the investment strategy adopted.

3.3.2 Several stochastic asset models have now been published since David Wilkie's original paper in 1986. Depending on the outputs that they produce (in particular with regard to the provision of zero coupon yield or price curves) vis a vis the sophistication of the economic reserving model that is desired (see 3.5.2 and 3.5.3 below) they may or may not be suitable for use in the current problem. Lee and Wilkie (2000) is a recent survey of the outputs produced by various published models.

3.4 *Investment strategy*

3.4.1 For each future year of projection, the investment strategy (i.e. the allocation to the asset classes supported by the stochastic asset model) must be specified. This could be implemented in several different ways, including:

- a static strategy, where the allocation is constant for each future year and regardless of model outcomes (e.g. solvency margins)
- a partially dynamic strategy, which although varying by time, is independent of future model outcomes

- a fully dynamic strategy, which varies according to both time and model outcomes.

3.5 *Reserving behaviour*

3.5.1 Assets and liabilities do not generally operate in a hermetically sealed container, without any external monitoring or corrective action being taken. Both legislation and principles of sound financial management require that regular estimates be made of the reserves that are required to meet outstanding liabilities.

3.5.2 Reserving models in the context of annuity business may be split into two different areas: economic and mortality reserving models. The role of the economic reserving model is to supply the assumptions that will be used to project (e.g. via the provision of an assumption for future price inflation) and then discount future financial cashflows. In practice, this probably means providing at a minimum fixed income and inflation linked zero coupon price curves, with investment returns on equities and other assets being also necessary if there is any link between future benefits and investment performance (as for example with discretionary pension increases in a defined benefit pension plan, or if annuities contain any with profits or unit-linked element). The role of the mortality reserving model is to supply the assumptions that will be used to estimate future rates of mortality.

3.5.3 Once again, several different implementations are possible. For the economic reserving model, these include:

- at the simplest level (static basis), zero coupon price curves may be flat, and constant (at least in inflation adjusted terms) over the projection period. At the time of writing, such economic reserving models are still commonly used in the UK for the valuation of defined benefit pension scheme liabilities, although supplemented by additional calculations on non flat and non constant bases for discontinuance purposes, and more recently to meet the Minimum Funding Requirement
- at the most sophisticated level (fully dynamic basis), zero coupon prices will be both non flat, and varying over time with the economic conditions produced by the stochastic asset model
- it must be stressed that, although in many cases they may be, the reserving zero coupon prices need not be identical with the market zero coupon prices produced by the stochastic asset model as part of the normal output of such a model. For example, reserves may be set using an assumption of zero coupon rates plus a flat 0.5% p.a. to reflect the use of corporate bond yields, or may be lower than market rates, to reflect implicit margins for prudence which may in fact be required by legislation. In general, to increase comparability across different sets of liabilities, my own preference would be for the use of market rates together with explicit solvency margins (e.g. a requirement that assets should be at least 104% of liabilities).

3.5.4 For the mortality reserving model, possible implementations include:

- a static basis, under which a fixed expected mortality model, with or without allowance for future improvements, (see 3.2.2 above) is used to calculate the expected future rates of mortality
- a dynamic strategy, under which the expected mortality model is reviewed at period intervals, in the light of stochastically generated experience.

Note that the expected mortality model used for reserving need not be the same as that underlying the stochastic mortality model used to project actual mortality. In reality the true mortality model is unknown, and so is unlikely to coincide with the reserving model used. It is helpful therefore if the design of the system allows at the outset for a situation where for example, reserving is carried out using PA(90) with an age rating (the basis currently prescribed both for Minimum Funding Requirement and for Pension Review Loss Assessment purposes), but actual mortality is based around more realistic expectations of PMA92 / PFA92 or a cohort model.

3.5.5 If the mortality reserving model is dynamic, rules need to be specified as to how the expected mortality model used for reserving changes with time, and in the light of experience. (Note that the experience taken into account does not necessarily have to be that of the population under consideration, but could be that obtained from a larger and more statistically significant population, e.g. the experience of the life office as a whole. The system needs to allow for such a possibility, and also in such a situation for the larger population to be modeled at the same time, so that its experience is available!).

3.5.6 The experience adjustment rule that I have adopted for the purposes of this paper is a simple credibility theory approach, as follows. Mortality experience is reviewed every n years, where n is an input parameter. An experience adjustment ratio vector $r_1(t)$ records the total Actual to Expected ratio for deaths, using exposure by amounts, at the most recent mortality experience review on or before year t. An actual adjustment ratio factor $r_2(t)$ is calculated as follows:

$r_2(t) = a * r_1(t) + (1-a) * r_2(t-1)$ if $t = 0 \bmod n$ (i.e. t is an experience review year)

$r_2(t) = r_2(t-1)$ otherwise

where a and $r_2(0)$ are input parameters.

The expected mortality model used for reserving at time t ($t>=0$) is then adjusted from the expected mortality model used for reserving at time 0 **before** adjustment for $r_2(0)$ by multiplying the **base** q_x factors (e.g. q_x for PA(90) or q_x for PMA92Base / PFA92Base) by $r_2(t)$. Note that multiplying all the future "q_xs" (i.e. the one year probabilities of death) by $r_2(t)$ does not mean that probabilities of death over periods longer than one year (${}_tq_x$) are also multiplied by the same ratio.

3.5.7 Suggestions for alternative implementations of experience adjustment rules will be gratefully received. This model applies a flat proportional adjustment up or down to death rates at all ages, and is therefore a significant over-simplification. However, my objective was to move from a 0th order situation, where no experience adjustment was applied, to a 1st order situation where, even if differences in the experience by age were ignored, the overall impact looked correct to 1st order. On the other hand, simply to adjust each q_x by the A/E ratio at that age might produce mortality rates that were unrealistic in their lack of smoothness, so that option was rejected. At the other end of the scale, an nth order solution would be to model the calculations carried out by the CMI, which include curve fitting by non linear optimisation, together with judgement in order to reject curves produced by unconstrained optimization when they produce curves that, whilst mathematically plausible, are simply not credible as representing mortality rates. Such a process would not only be extremely time consuming computationally as part of a stochastic process, but difficult to model because of the judgemental part required.

3.5.8 For reasons similar to those given in 3.2.5, I suggest that the model described in 3.5.6 and 3.5.7 be called InQA mortality reserving model 1.

4 Specimen Results

4.0 In this part of the paper I show examples of the types of analyses that can be carried out using the framework that I have developed. Since a large number of submodels is involved, for each of which there are parameter variations to be considered also, the number of possible combinations of results that could be shown is extremely large. The results shown are presented as indicative specimen results, rather than intended for detailed analysis at this stage.

4.1 Although the framework is of course applicable to more complex liabilities, I have taken a simple example as a basis for these initial specimen results, namely a portfolio of male lives aged 60 to 69 with annuities increasing in line with Limited Price Indexation (LPI, i.e. price inflation with a maximum of 5%, but with the proviso that pensions "tread water", ie never fall in value, but wait until the LPI index has caught up with its previous maximum value, in the event of negative price inflation). For the purposes of this simple illustrative example, I have ignored spouses' pensions and have projected only 100 simulations. (Note: to obtain more accuracy with regard to percentiles, practical work shows that over 1,000 simulations and preferably 10,000 simulations are required.)

4.2 Table 4.1 shows the initial portfolio of liabilities as at an initial valuation date of 31 December 1999. I then project the portfolio of business forward using a variety of submodels, including:

- stochastic asset models: Wilkie (1995), Smith Jump Equilibrium (Smith 1996), Cairns (1999a and b). In order to produce zero coupon prices z_T for the Wilkie model, I have used the simple par yield curve formula described in Lee and Wilkie (2000), namely g_T = par yield for term T = C(t) + {B(t) – C(t)} exp (-kT), from which zero coupon prices may be obtained by recursion using the formula $z_T = (1- g_T \Sigma_1^{T-1} z_i) / (1 + g_T)$. (NB I have simply used published parameters for each model at this stage, and have not attempted to recalibrate the models to use consistent parameters, which is in any case a non trivial task.)

- economic reserving models: I show both a sophisticated dynamic zero coupon price basis (taking the actual stochastic yield curves produced by the relevant asset model), and the simplistic constant flat yield curve approaches (I take a flat 9% p.a. discount rate and 5% p.a. inflation basis, which has been a widely used flat actuarial basis for pension fund valuation purposes in the UK)
- actual mortality: in each case I use InQA mortality model 1 (see 3.2.5), with parameters $\sigma_X = 0.0075$ and $\sigma_Y = 0.0375$. I chose these values by comparing the actual experience A/E for combined ages 61-100 (in Table PEN 1.4 page 74 of CMIR 1998) with simulations of experience using mortality model 1. The simulations were centred around a trend line fitted using Microsoft Excel's linear regression facility, and graphs were plotted of simulated experience against actual (CMIR) experience. I found that, on the assumption that $\sigma_Y = 5*\sigma_X$ (a working suggestion of David Wilkie's), values of σ_X which were outside a range of 0.005 to 0.01 produced simulated experiences which looked markedly different from the actual experience, because they looked significantly less / more random than actual experience respectively.
- mortality reserving: in each case I use InQA mortality reserving model 1 (see 3.5.8), but I show the impact separately of a) having a reserving basis different to the actual mortality basis, and b) allowing for adjustments in the light of experience.

4.3 I draw the following (extremely tentative) conclusions (given that only 100 simulations have been carried out) from the calculation results shown in Tables 4.2, 4.3 and 4.4:

- At present, PA(90) M rated down 2 years implies reserves which are significantly lower (of the order of 10-30%) than PMA92Base projected forward. This is not surprising, and echoes the previous findings of CIMR (1999) and Willets (1999).
- In the situation shown (where the mortality reserving basis differs significantly from the unknown by true mortality basis) the impact of allowing for mortality experience on an annual basis with a credibility weighting of 0.5 leads to reserves within 5 years which, for a given set of economic assumptions, are much closer to the true reserves required (i.e. those shown in the first three columns of Table 4.3)
- Table 4.4 shows that, for the population sizes shown (with in each case around 100 lives at each age attained at the initial valuation date), the impact of stochastic mortality (i.e. of having non zero σ_X and σ_Y in InQA mortality model 1) is relatively small, in each case altering the liability reserves (and their interquartile ratio) by less than 0.25%. However, the impact of not knowing the true mortality basis can be highly significant, as shown by a comparison between the reserves for the first 3 columns for Tables 4.2 and 4.3.

Table 4.1 Initial Liability Data, as at 31 December 1999

Age	TotalPensions (£000 p.a.)	AveragePensions (£000 p.a.)
70	1000.00	10.00
71	1050.00	10.00
72	1100.00	10.00
73	1150.00	10.00
74	1080.00	9.00
75	1010.00	8.50
76	940.00	8.00
77	870.00	7.50
78	800.00	7.00
79	730.00	6.50

Table 4.2 Impact of allowing for mortality experience adjustment (projections over 5 years)

MODELS USED						
Investment Strategy	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL
Asset	Wilkie	SmithJE	Cairns	Wilkie	SmithJE	Cairns
Economic Reserving	Dynamic zeros	Dynamic zeros	Dynamic zeros	Dynamic zeros	Dynamic zeros	Dynamic zeros
Actual Mortality	InQA1 PMA92	InQA1 PMA92	InQA1 PMA92	InQA1 PMA92	InQA1 PMA92	InQA1 PMA92
Mortality Reserving (Expected Mortality)	InQA1 PA(90)M-2	InQA1 PA(90)M-2	InQA1 PA(90)M-2	InQA1 PA(90)M-2	InQA1 PA(90)M-2	InQA1 PA(90)M-2
Mortality Reserving (Experience Adjustment)	none	none	none	every year, a=0.5, r2(0)=1.0	every year, a=0.5, r2(0)=1.0	every year, a=0.5, r2(0)=1.0
RESULTS (£000s)						
Initial Assets	80000	80000	80000	80000	80000	80000
Initial Reserves	71631	73551	79226	71631	73551	79226
Final Assets (deterministic)	58997	63845	50835	58997	63845	50835
Final Reserves (deterministic)	58115	57780	52644	71326	71456	65606
Final Assets 25%ile	57900	51121	46873	57900	51121	46873
Final Assets 50%ile	61089	69923	51461	61089	69923	51461
Final Assets 75%ile	65214	90757	55421	65214	90757	55421
Final Reserves 25%ile	51717	53787	50754	63503	65789	62875
Final Reserves 50%ile	53856	57610	52759	66248	70918	66038
Final Reserves 75%ile	56433	61217	54661	68950	76780	68224

Table 4.3 Impact of a) more accurate mortality reserving and b) more naïve economic reserving (projections over 5 years)

MODELS USED						
Investment Strategy	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL
Asset	Wilkie	SmithJE	Cairns	Wilkie	SmithJE	Cairns
Economic Reserving	Dynamic zeros	Dynamic zeros	Dynamic zeros	Flat 9/ /5 basis	Flat 9/ /5 basis	Flat 9/ /5 basis
Actual Mortality	InQA1 PMA92	InQA1 PMA92	InQA1 PMA92	InQA1 PMA92	InQA1 PMA92	InQA1 PMA92
Mortality Reserving (Expected Mortality)	InQA1 PMA92	InQA1 PMA92	InQA1 PMA92	InQA1 PA(90)M-2	InQA1 PA(90)M-2	InQA1 PA(90)M-2
Mortality Reserving (Experience Adjustment)	none	none	none	none	none	None
RESULTS (£000s)						
Initial Assets	80000	80000	80000	80000	80000	80000
Initial Reserves	85026	87660	95317	72399	72399	72399
Final Assets (deterministic)	58997	63845	50835	58997	63845	50835
Final Reserves (deterministic)	69288	69299	63516	58629	55909	48858
Final Assets 25%ile	57900	51121	46873	57900	51121	46873
Final Assets 50%ile	61089	69923	51461	61089	69923	51461
Final Assets 75%ile	65214	90757	55421	65214	90757	55421
Final Reserves 25%ile	61497	63824	60746	52261	53997	48015
Final Reserves 50%ile	64142	68928	63446	55364	56343	49463
Final Reserves 75%ile	67263	73878	65862	57143	57698	51615

Table 4.4 Comparing impact of stochastic versus deterministic actual mortality (projections over 5 years, first 3 columns repeat last 3 of Table 4.3 for ease of reference)

MODELS USED						
Investment Strategy	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL	50/50 FI/IL
Asset	Wilkie	SmithJE	Cairns	Wilkie	SmithJE	Cairns
Economic Reserving	Flat 9/ /5 basis	Flat 9/ /5 basis	Flat 9/ /5 basis	Flat 9/ /5 basis	Flat 9/ /5 basis	Flat 9/ /5 basis
Actual Mortality	InQA1 PMA92	InQA1 PMA92	InQA1 PMA92	deterministic PMA92	deterministic PMA92	deterministic PMA92
Mortality Reserving (Expected Mortality)	InQA1 PA(90)M-2	InQA1 PA(90)M-2	InQA1 PA(90)M-2	InQA1 PA(90)M-2	InQA1 PA(90)M-2	InQA1 PA(90)M-2
Mortality Reserving (Experience Adjustment)	none	none	none	none	none	none
RESULTS (£000s)						
Initial Assets	80000	80000	80000	80000	80000	80000
Initial Reserves	72399	72399	72399	72399	72399	72399
Final Assets (deterministic)	58997	63845	50835	58997	63845	50835
Final Reserves (deterministic)	58629	55909	48858	58629	55909	48858
Final Assets 25%ile	57900	51121	46873	57857	51071	46917
Final Assets 50%ile	61089	69923	51461	61081	69853	51471
Final Assets 75%ile	65214	90757	55421	65217	90819	55391
Final Reserves 25%ile	52261	53997	48015	52168	54125	47932
Final Reserves 50%ile	55364	56343	49463	55429	56377	49371
Final Reserves 75%ile	57143	57698	51615	57163	57748	51726

5 CONCLUSIONS

5.0 Object-oriented, polymorphic frameworks such as the one outlined in this paper offer powerful tools to analyse a wide range of asset liability management problems. I have described two new (to my knowledge) but simple (and therefore possibly too simplistic) models, one (suggested to me originally by David Wilkie) for stochastic mortality (InQA stochastic mortality 1 -see 3.2.5), and one for allowing for the updating of mortality reserving in the light of recent experience (InQA mortality reserving 1 – see 3.5.7).

5.1 I invite comments and criticisms on the framework, these new models, suggestions for alternative and better models, and for the areas which would be of most interest to others with regard to future calculations and other analysis.

5.2 Acknowledgements: I am grateful to David Wilkie for helpful comments at various stages during the preparation of this paper. The responsibility for any errors or omissions in this paper lies with me alone, however.

REFERENCES

CAIRNS, A.J.G. (1999a) A multifactor equilibrium model for the term structure and inflation. *Proceedings of the 9th International AFIR Colloquium, Tokyo*, 93- 113.

CAIRNS, A.J.G. (1999b) A multifactor equilibrium model for the term structure and inflation for long-term risk management with an extension to the equities market. *Unpublished technical note 99/19*, Heriot-Watt University, Edinburgh.

CMIR (1990) (Standard Tables of Mortality based on the 1979-1982 Experiences). *Continuous Mortality Investigation Reports, Number 10*.

CMIR (1998) (Various Reports on the 1991-1994 Mortality Experiences). *Continuous Mortality Investigation Reports, Number 16*.

CMIR (1999) (Standard Tables of Mortality based on the 1991-1994 Experiences). *Continuous Mortality Investigation Reports, Number 17*.

LEE, P.J., WILKIE, A.D. (2000) A comparison of stochastic asset models. (forthcoming, *2000 Investment Conference of the Institute and Faculty of Actuaries*.)

SMITH, A.D. (1996) How actuaries can use financial economics. *British Actuarial Journal*, **2**, 1057-1174.

WILLETS, R. (1999) Mortality in the next millennium. Presented to the *Staple Inn Actuarial Society*, 7 December 1999.

WILKIE, A.D. (1986) Some applications of stochastic investment models. *Journal of the Institute of Actuaries Students' Society*, **29**, 25-51.

WILKIE, A.D. (1995) More on a stochastic asset model for actuarial use. *British Actuarial Journal*, **1**, 777-964.