JUST HOW RISKY ARE EQUITIES OVER THE LONG TERM?

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(1) INTRODUCTION

(1.1) It is generally recognised that equities are riskier than the other main assets (such as cash, bonds or even property) over the short term. However, there is a strong view that the relative risk diminishes with time so that for long-term investors, such as pension funds or life offices, equities can be viewed as almost as safe as cash if held for a long period, yet offering a higher real return.

(1.2) I shall show that this intuitive view is indeed correct, but only for a “long term” that is longer than most people probably currently think, and as a consequence for the vast majority of investors the days of 100% equity asset allocation have not yet arrived. Instead, the old principle of holding a portfolio well diversified across the major asset classes (including overseas equities, even if the liabilities of the investor are sterling denominated) still holds true. The asset allocation that is best suited to any particular investor is best determined by an asset liability modelling study which makes full allowance for:

- the particular liabilities of the investor
- the effects of volatility over the long term

(1.3) I shall show that some of the effects of volatility are surprising: for example there is a skewness effect that might be termed a “volatility bias” if one considers traditional mean variance optimisation (also known as “efficient frontier analysis”) over long time periods. By this I mean that for two assets with the same mean continuously compounded rate of return but different volatilities (standard deviations), over any period £1 invested in the higher volatility asset will have the higher mean accumulation, and this effect actually increases with the time period. This means that mean variance optimisers will select the higher volatility asset in preference to the lower volatility one, which is a result that will appear counter-intuitive to many readers. Of course, mean variance optimisation is only a convenient artifice to narrow down the range of possible optimal portfolios. To find the true optimal portfolio for an individual investor one would need to know the utility function for that individual (a process that may not be possible!). Thus, it may be that mean variance optimisation is inappropriate in this context.

(1.4) As an alternative to holding a well diversified portfolio spread across the major asset classes, I explore two alternative strategies also aimed at containing the risks of equity investment while attempting to retain the benefits of their higher expected return. These are so called “portfolio insurance” (which I shall show has not lived up to its name, neither can it be expected to in the form most commonly practised) and (more sensibly, but at non negligible cost and with implementation problems) the strategy of protecting equity capital values with put options.

(1.5) To avoid giving the reader unnecessary indigestion, the mathematical derivations of the results presented in this paper have been relegated to the Appendix.

(1.6) I am grateful for comments from various colleagues. My thanks are due also to colleagues at Ringley Communications for their forebearance in transferring my handwritten scrawl to the printed page. Any errors, omissions, inaccuracies or controversial opinions in this paper remain entirely my own responsibility, however!
(2) HOW LONG DOES THE LONG TERM HAVE TO BE?

(2.1) Let me start by posing a question:
How long must an investor be prepared to wait to be reasonably confident that equities will beat inflation?

Historical Experience

(2.2) This is a natural starting point. Taking the period 1919-1989 as the longest period that I would feel comfortable with as representative of modern investment conditions (some would prefer to look at shorter periods), there has been only one 20 year period over which the return on UK equities has failed to keep up with inflation and that is the period 1955-1974. What does this tell us?

(2.3) First of all, it reminds us that equities can perform abysmally, which is something that is all too easy to forget after the superb returns experienced in the 1980s. Secondly, it is tempting to argue that as there are 52 different 20 year periods in 1919-1989 (starting from the period 1919-1938, moving to 1920-1939, etc and ending with 1970-1989) the probability of a negative real return on equities over 20 years must be of the order of 1/52, or just under 2%. However, this reasoning is suspect because the 52 20 year periods are not independent, but overlap.

(2.4) How about arguing that we have about three and a half independent 20 year periods in 1919-1989, so estimate the probability of negative 20 year real returns at 1/3.5 or about 29%? The trouble here is that three and a half doesn’t give us much of a sample to go on!

(2.5) So, 2% or 29%? I think the answer lies, as you would expect, somewhere in between, as I hope the following statistical analysis will demonstrate.

Statistical Modelling of Equity Real Returns

(2.6) Having exhausted the empirical evidence, we need to turn to statistical theory for some answers. In what follows, I examine the implications of three possible models for equity real returns. Of course, there are many possible models that could be tried, but they are likely to be very similar to one of my three general models, so the broad validity of my conclusions will still hold.

(A) Independence Model

(2.7) In this model, the real rate of return in one period is statistically independent of the rates of return in all previous periods. This is what one would expect if the weak form of the efficient market hypothesis (EMH) is true: the market has already discounted the information contained in the history of previous returns.

(2.8) For example, if a poor return one year had a strong tendency to be followed by a “catching up” good return in the next year and vice versa (ie the negative autocorrelation model on page 8), then because investors will be aware of this tendency, most investors will want to buy stocks that have underperformed in the last year. Very few investors will be
willing to sell such stocks at their current, low price, so the price of such stocks will be bid up very quickly until it reaches a price at which sellers will equal buyers. This equilibrium price will be such that the return over the past year if recalculated will now be found to be only average!

(2.9) There is strong support for the validity of the weak form of the EMH (see reference 1) in relation to stock selection (ie one should not expect to be able to predict on the basis merely of past share prices and dividends which of say ICI or Glaxo is likely to perform better over the next year).

(2.10) What is less clear is to what degree the EMH is valid in the context of asset allocation (eg predicting whether a particular equity index will outperform a particular bond index over the next year). However, the distinction between stock selection and asset allocation is somewhat artificial in my view. High yield bonds can behave very much like equities for example and one might view a particular gilt and a brewery stock to be no more dissimilar than the same brewery stock and a property investment company, say.

(2.11) It is the activities of the equity investment analysts which make the weak form of the EMH valid for equities. Bond investment analysts similarly ensure weak EMH validity in the bond markets. The extent to which the weak EMH is invalid for asset allocation must reflect the difficulties of synthesising the results of analysis across different asset classes.

(2.12) There is evidence that this type of inefficiency has been present in the past, at least in the UK, as shown by the Wilkie model which is a non independence model incorporating features of both models B and C below, fitted from UK data over the period 1919 to 1982 (see reference 2). This "asset allocation" inefficiency appears to have persisted since 1982 in that the model continues to provide a good fit.

(2.13) How long this can be expected to continue is a matter for subjective judgment. I suggest that it is both a challenge and an opportunity (for those who get in first!) to investment managers to remove this inefficiency as far as possible!

(2.14) What is a suitable distribution for the return (either on a market value or on an assessed value basis) in a particular period? Well, there is evidence that continuously compounded security returns (akin to the "force" of interest δ which accrues continuously, rather than the rate of return r that has been earned at the end of the year) are (at least over short periods) approximately normally distributed. Let us say that the mean and standard deviation of this normal distribution for the continuously compounded annual real rate of return on equities are μ and σ respectively.

(2.15) If we take our equity portfolio as well diversified not only across industrial sectors but also internationally (to obtain the maximum benefit from diversification: the maximum reduction in σ), most experts would currently put the parameters μ and σ for the real annual return to a UK investor in the ranges:

\[
\begin{align*}
\mu & : \quad 5\% \text{ to } 7\% \\
\sigma & : \quad 15\% \text{ to } 25\%
\end{align*}
\]

(For an informative guide to historical returns, I refer the reader to the BZW Equity-Gilt study, published annually).

(2.16) I shall initially take μ = 6% (strictly speaking, the log of 1.06 for technical reasons) and σ = 20%.
In rough terms, this means that the annual real return on equities can be expected to have a two thirds probability of lying in the range -14% to +26% (i.e. 6% ± 20%). One sixth of the time it can be expected to lie below -14% and one sixth of the time it can be expected to lie above +26%.

(2.17) Graph 1 shows the 5th, 50th and 95th percentiles as well as the mean of the annualised return looking at investment in the equity portfolio for periods of up to 50 years.

(2.18) The pth percentile is the point below which the return will lie with probability p%. Thus the 5th and 95th percentiles can be regarded loosely as “worst” and “best” cases at the 5% probability level.

(2.19) You can see from this graph that, to borrow a phrase from Redington, when one considers returns, there is a narrowing funnel of doubt. Over longer periods, the median return is increasingly likely to be achieved (and the mean return eventually becomes the same as the median return, although it is initially higher).

(2.20) Graph 2 shows the same statistics for the value of the portfolio invested in equities (arbitrarily started at 1). You will note that in this case, and this is very important, there is an expanding funnel of doubt. Focusing on returns is actually very misleading because what matters in meeting liabilities is not the mathematical abstraction of returns but the physical reality of how much money is available to meet those liabilities. The graph shows that far from settling down over time, the variability of equity portfolio values actually increases!

(2.21) Whether this result is equally true whether one is considering market values or assessed values, I submit for discussion. My view is in the affirmative since I would see the model as equally applicable to market or assessed value returns.

(2.22) Graphs 3 and 4 show parts of graph 2 in greater detail.

(2.23) Returning to our original question, you can see that you have to wait at least 31 years to be confident at the 5% level that you will beat inflation by investing in equities. (This is the point at which the lower line, i.e. the 5th percentile, moves above 0 for graph 1 or 1 for graph 2). In fact, further analysis shows that if you do have a liability horizon of 31 years, there is still a 2.5% probability that you will have lost at least 31% of your capital in real terms by the end of the 31st year! If you want to be confident of beating inflation at the 2.5% probability level, then you need to wait at least 46 years. What if we change the parameters?

(2.24) If you feel that 6% is too optimistic for a real rate of return and would prefer 5% but with lower volatility (say 15%), then the figures of 31 years and 47 years become 26 and 37 respectively. With \( \mu = 7\% \) and \( \sigma = 25\% \) they become 37 and 53 years.

(2.25) The point is that under this model with most reasonable settings of \( \mu \) and \( \sigma \) there is at least a 5% probability that your equity portfolio will fail to beat inflation over a 20 year period. In fact the probabilities under the above settings for periods of 10 and 20 years are:

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( \sigma )</th>
<th>10 year Prob. of loss</th>
<th>20 year Prob. of loss</th>
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<td>5</td>
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Most trustees and finance directors (the latter in particular) would feel most uncomfortable with such high risks of loss (although the use of assessed values will mitigate these losses to a degree). It is all very well arguing that unless the scheme is fully mature, the term of the liabilities will extend beyond 20 years:

- in practice, the pain of sustained losses over even 10 years would be too great (the finance director will have even shorter time horizons)
- the figures above show that even a 40 year term may not be long enough
- under the independence model, if the first 10 years turn out to be bad, then there is still no reason to expect subsequent returns to be any higher than average!

Under what sort of circumstances could a well diversified international equity portfolio fail to keep up with inflation over a long period? Here are a few possible scenarios:

- widespread fear of, but not actual, collapse in the banking system
- a resurgence of political risk involving anti free enterprise policies
- a climate of sustained stagflation combined with increasing protectionism (collapse of GATT talks?)
- a major protracted war
- massive pollution clean up costs imposed on firms by governments
- discovery of large scale fraud within several large multinationals
- widespread adoption of fashionable, but flawed management techniques.

Positive Autocorrelation Model

Under this model, if past returns have been better (or worse) than average, then the return in the next period is likely to be a little worse/better (ie there is a tendency to revert to the mean), but still better/worse than average.

This would be a clear violation of the weak efficiency hypothesis with regard to asset allocation, for which there is some evidence with regard to the past, but which I would expect to decline in future years as investment managers take advantage of this inefficiency.

However, if you believe that such asset allocation inefficiencies are likely to persist, then the effect of positive autocorrelation is to significantly increase the volatility of returns and portfolio values over periods longer than one year.

The effect of such higher volatility is to increase the probability of loss drastically. In fact, if one supposes that the best estimate of the excess return (ie the return above the mean, whether positive or negative) in the next period is 60% (a typical figure for such models) of the excess return last period (under the independence model the best estimate would be zero), then one has to wait for 83 years to reduce the probability of loss below 5%, even with the most favourable settings (within the range above) of \( \mu \) and \( \sigma \) at 7% and 15% respectively! See the mathematical appendix for an explanation of this fact. Equities become even riskier!
Apart from implying persistent market inefficiencies, another interesting feature of this model is that it is unstable. If it were true, then all investors would sell equities only after they had performed poorly and buy only after periods of good performance. This would push the prices of equities down to near zero after poor performance and would push prices through the roof after good performance, until eventually a panic would occur as the credibility of the theory became strained. Prices would shoot back up (down) again, but with rather fewer investors inclined (or able to given the likelihood that they will have got their fingers burnt) to subscribe to the theory. Thus if the model is correct, not all investors can be assumed to believe it to be correct - otherwise the model soon ceases to be correct!

(C) Negative Autocorrelation Model

Under this model, which has wide intuitive appeal, a below average return in one period is likely to be followed by "compensatory" above average returns in subsequent periods. It has been frequently said for example that the fantastic returns achieved in the 1980s were really a catching up exercise to make up for the poor returns in the 1970s.

Perhaps this is so, but I suspect that in 1974 or even in 1979 not many equity investors felt so confident that the very real losses on their portfolios were merely a piece of bad luck, a mistake that would be corrected in the 1980s! If the correction was expected to be made by "the market", then every investor must have been blaming every other investor for "undervaluing" his or her portfolio in the 1970s - the circularity in this is evident. (If the correction was expected to be made by some force other than the market, then one would have to believe in some sort of divine intervention to ensure with scrupulous fairness that for example the miners lost in the 1980s simply because they had won in the 70s!)

Like the positive autocorrelation model, the negative autocorrelation model also implies a degree of persistent inefficiency in asset allocation within the market, but has the advantage of being stable. It is possible for all investors to believe in it without the model breaking down due to prices being driven either through the roof or through the floor. Instead, prices would move alternatively up and down on a gentle roller coaster and the market would resemble a child playing repeatedly with an on-off switch.

This model might from time to time have limited validity if one believes that the market, although getting things right eventually, has a persistent tendency to overshoot leading to slight overvaluation of previously undervalued asset classes and slight undervaluation of previously overvalued classes.

Such persistent tendencies are likely to reduce in future with the fall from favour of portfolio insurance, highly leveraged takeovers and, more mundanely, a decline in the importance of the median pension fund asset allocation as a significant minority of pension funds set their own, liability driven, customised benchmarks. (See the section on asset allocation on page 10.)

Nevertheless, if you believe in this model, then the effect is to reduce volatility over long periods, by as much as $\frac{1}{2}$ (Again, see the appendix for a derivation). You may be tempted still further by the knowledge that if $\mu = 6\%$ and $\sigma = 20\%$, you only need a horizon time of eight years to beat inflation with a 95% probability...

This model seems to me to be overly optimistic.

Returning to reality:
(3) EQUITIES REMAIN VERY RISKY EVEN OVER THE LONG TERM!

(3.1) The above analysis shows that on the basis of evidence currently available, both historical and statistical, there is a high risk (of the order of 15%) that equities will not keep pace with inflation over 10 years, and a still painful risk (of the order of at least 5%) that your pensioners, policyholders or finance director will be extremely uncomfortable even after 20 years. If anyone has a model that produces more palatable figures (a chaotic model perhaps?), I would be glad to hear from them!

(3.2) Amidst such doom and gloom, what can be done about this? Well, firstly, most investors will have already taken positive steps.

(3.3) What the above analysis shows, is that a portfolio consisting entirely of UK equities is likely to be too risky even for the most immature pension funds or the richest funds (the ones with the largest surpluses) because the volatility will be unnecessarily high. It can be reduced significantly (from around 20% to possibly 15%) simply by the inclusion of overseas equities in the portfolio.

(3.4) Who should hold a portfolio consisting entirely of international equities, then? Well firstly the richest investors whose first choice would have been the 100% UK equity portfolio: research has shown that they can reduce volatility (and hence the risk of loss) without any loss in expected return. However these investors would need to be rich enough to bear possibly up to a 30% capital loss in real terms over a 20 year period with a non negligible probability.

(3.5) The rest of us will need to follow the conventional wisdom of diversifying across the whole range of major asset classes and include bonds (including international bonds), cash and property. (For very large funds, according to some recent American studies, gold and other non perishable commodities may be worthy of consideration!)

Why does the conventional wisdom make sense? The answer lies in the reduction in the volatility parameter \( \sigma \) that arises because of the combination in the same portfolio of asset classes that display little correlation with one another. Such diversification really does smooth out the extremes of performance, with a poor performance in one asset class being likely to be compensated to some extent by a strong performance in other classes (and unfortunately vice versa: the other side of the coin is that the upside is also reduced!).
(4) ASSET ALLOCATION

(4.1) It is comforting to know that the age old proverb about not putting all your eggs into one basket remains just as true in the era of the rocket scientists! Unfortunately the proverb stops short of telling us how many eggs to put into each of these baskets.

(4.2) To determine the asset allocation that is most appropriate for your own circumstances and liabilities, there are at least four common approaches:

(4.2.1) You can trust in the collective wisdom (or unfortunately sometimes otherwise) of the marketplace - you simply try and follow the asset allocation that your peers are adopting. This has the tremendous virtue of being cheap. On the debit side:

- who is to lead the process?

- this is an essentially unstable process if widely practised. (Recent years have luckily not demonstrated this to the full. In theory prices can be driven to unsustainable heights and depths in this way and this may partly reflect what happened in the summer and autumn of 1987.)

- the market professionals to whom the day-to-day investment responsibilities are delegated are of necessity distracted from their primary task (to add value by thorough research enabling them to make tactical asset allocation and stock selection gains) by the time consuming need to be constantly looking over their shoulder to guess what allocation the other managers are adopting. (To wait until the information becomes publicly available could mean too long a delay.)

- the median distribution is a moving target

- the average allocation may be fundamentally unsuitable for your fund

Thus there are undoubtedly some heavy hidden costs here.

(4.2.2) You may improve on the above by moving to a fixed target benchmark allocation, simply by taking for example the average CAPS median distribution over the last three years say. This would be reviewed every three years and removes some of the disadvantages of the first method whilst offering the advantages of:

- cheapness (again, there will be hidden costs through the disadvantages which I shall mention below)

- the "herd" instinct is broken - because the benchmark is fixed, if a 1987 type upward gyration of equity values occurs, the proportion in equities will become too high, leading to progressively more and more sales of equities (to reduce the proportion) as the market rises; conversely, as prices fell, more and more equities would be bought. Thus the fund would be selling at the peaks of markets and buying at the troughs, which is intuitively the more sensible approach for a long-term investor.

- the professionals are free to do their job rather than crystal ball gazing in order to predict the contents of the next CAPS quarterly report.
The big disadvantage that remains is that the benchmark has still been chosen in a fairly arbitrary manner and may not be suitable for your fund. But it is, nevertheless a vast improvement on the first method.

(4.2.3) An asset driven asset allocation exercise may be undertaken. This sort of exercise uses very little information about the fund’s liabilities other than qualitative information of the type “the liabilities are basically real in nature - the absolute lowest real rate of return that we could tolerate over the term of the liabilities is y%” in conjunction with a quantitative analysis of the assets, involving sophisticated mathematical and statistical techniques. In this way the vast field of possible candidates for a strategic asset allocation benchmark can be narrowed down to a shortlist that makes a lot of sense from the asset side and at least some sense on the liability side.

The advantages are obvious, at least some of the hidden costs involved in the two previous methods will be removed by adopting this more measured approach.

The disadvantages are:

- a feeling that somehow this is only a “half way house solution” that falls short of taking full account of the liabilities

- less obviously there are some technical shortcomings to do with the long-term nature of the liabilities and which are usually ignored by purely asset based studies:

  a) we saw above that focusing on returns over long periods gives a misleading understatement of the risks. It is the value (whether assessed or market) of the portfolio that matters when meeting liabilities and that is much more volatile (see graphs 1 and 2 again).

  b) graph 5 shows another counter intuitive result. Over the long term, more volatile assets will tend to outperform less volatile ones. This very real feature (see the appendix for a proof) will be present in a long-term liability driven study but becomes invisible when the analysis is distorted by looking at returns (indeed it is practically undetectable when only one year returns are studied, which has so far been the norm for such studies).

(4.2.4) The ideal approach is for an asset liability modelling exercise to be carried out, taking account of the liability cashflows in full. These exercises involve the calculation of a large number of actuarial valuations under varying investment conditions, so the full interaction of the future portfolio values and liability benefit payments can be measured. By comparison with the third method above, the process is quantitative both with respect to the assets and the liabilities, and it takes explicit account of the long-term effects. For this reason the trustees or sponsoring company can be presented with the likely consequences under varying scenarios of adopting particular investment policies in terms of the scheme’s funding level (which trustees have particular interest in) or in terms of the company contribution rate (of most interest for the sponsor) at a horizon date in the future. It is difficult to see how such information could be provided without undergoing an asset liability modelling exercise. (See the papers by Wise, Wilkie and Lockyer for a description of the techniques that are available (references 3, 4 and 5)).

(4.3) I conclude the paper with a discussion of two modern alternative approaches to the conventional “eggs in several baskets” wisdom: “portfolio insurance” and protected puts.
"PORTFOLIO INSURANCE"

(5.1) The idea is to protect yourself against market falls without buying options, by selling progressively more equities into cash as the market price falls. Thus by the time the price reaches a preset floor, you hold only cash and are thus insulated from further market falls.

(5.2) This seems (with justification in my opinion) to have been discredited since the 1987 crash when the effect in the USA was likened to a herd of stampeding bulls all trying to get through a narrow mountain pass at the same time! To illustrate, let's say that your current wealth is 100 and that you want to make sure that your wealth does not fall below 75. Initially the UK All Share Index stands at 1000, say. Under portfolio insurance you might decide to hold twice the excess of your wealth over 75 in equities. Thus initially you hold 50 in equities and 50 in cash. If the index now falls to 900, then your total wealth is now only 95, so under the portfolio insurance rule you now want to hold only 40 in equities, ie you sell 5 worth of equities so your portfolio becomes 40 in equities, 55 in cash.

(5.3) Providing the index does not suddenly drop massively, you will probably be able to keep selling in this way to ensure that at all times your wealth remains above 75. However, there are some serious flaws:

(5.4) Firstly, if the index suddenly collapsed to below 450, say 440, then even assuming you could sell at 440 your wealth would only be 74.56. In practice even if the index fell to 460 or 470, you would have difficulty selling at 450 because if portfolio insurance is the current fashion, (as was the case in the US in 1987), sellers will outnumber buyers at 450, so the bid offer spread will widen the price you obtain down to below 450.

(5.5) Secondly, if the index rose from 900 back to 1000, you would now have total wealth equal to 99.44, a loss of .56 even without allowing for transaction costs. Why has this occurred? Well, the portfolio insurer has to sell more and more as the market falls, and buy more and more as the market rises. In volatile markets (such as the world stock markets increasingly are), this is a recipe for disaster.

(5.6) In fact, the process not only fails to guarantee capital protection, but is itself unstable and thus itself likely to lead to the very capital loss that it is supposed to prevent. If many investors adopt portfolio insurance, then if markets are going down, the selling pressure from the portfolio insurers will drive them yet lower, thus leading to no end in sight until the portfolio insurers either abandon their strategy or accept massive losses. If on the other hand, markets are currently bullish, then the insurers will be driving the markets even higher by their need to buy yet more equities, until eventually portfolio insurers will hold almost 100% of their assets in equities, at prices which look increasingly unrealistic. The result? Eventually confidence will collapse and so will portfolio insurers’ money with the inevitable crash.
(6) PROTECTED PUTS

(6.1) In fact, portfolio insurers are keen free lunchers: they wish to have their cake and eat it by believing that they can reduce the considerable risks involved in equities (as I hope this paper will have demonstrated) with only a minimal reduction in expected return.

(6.2) As you would expect, it is in fact possible to pay for the privilege of genuine insurance, namely by purchasing put options to protect the capital value of your equity portfolio.

(6.3) A put option gives the owner the right (as opposed to the obligation) to sell a security at any point during a time period (usually several months) at a fixed price. The put owner thus knows that however low the market price, he or she will always be able to sell at the guaranteed price. The guarantee is provided by the options market exchange. There is in principle no need to rely on the financial soundness of the individual investor on the other side of the bargain, although in theory if the exchange has set insufficiently high margin requirements it is possible for the exchange itself to be forced to default in the event of large scale defaults on the part of individual investors.) Now surely the cost of such options in a free market should give us an indication of the true cost of providing real insurance against market falls, and the corresponding risks involved in equity investment.

(6.4) As an example, at 29 November 1990, the FTSE stood at 2138. One could have bought about 10 months’ capital protection for this index at the level of 2125 by purchasing the September 1991 2125 Euro-FTSE put option at 107 (probably 110 after transaction costs) or an annualised cost of the order of 6% per annum.

(6.5) In effect, by giving up about 6% return per annum, you will have transformed your extremely dangerous insufficiently diversified UK equity portfolio into some kind of superexciting cash asset, with limited downside but still unlimited upside. Following the 1990 Budget, pension funds are now allowed to invest without tax penalty in options and futures and this is an example of proper use of these instruments to reduce risk. There are three caveats:

(6.5.1) Beware tracking error: your UK equity portfolio is unlikely to match any index precisely (unless you have employed an indexed manager you should be unhappy if it does, as you will then be paying over the odds for effectively a closet indexed fund). Therefore, it is possible under certain circumstances (quantitative services would be able to tell you the sort of risks you are exposed to - maybe a fall in the oil price if you are overexposed to oil stocks) for your portfolio to fall while the index rises, thus rendering your put worthless.

(6.5.2) The terms of put option contracts are still very short (although in Chicago they have introduced two year contracts) so you would need to keep rolling over your put position by buying successive new options.

(6.5.3) You may well be justified in feeling that the put options are just too expensive - who in their right mind would give up 6% per annum on an equity portfolio that is expected to yield about 6% per annum? Have the markets gone crazy? Surely the put options are overpriced at 6%? The answer is that focusing just on returns (6% - 6% = 0%) is a narrow minded one-dimensional approach to a two-dimensional (three if you include the time factor) problem - volatility is the key. With limited downside, far from being a disadvantage, volatility is now a major asset. Look again at the means and 95th percentiles (the upside) in graphs 1 and 2 - the upside for cash will be very pedestrian in comparison.
(6.6) I conclude by saying that there are very tight bounds within which the relative prices of puts, calls (options which give the right to buy at a fixed price) and the underlying security can move, without risk free profits becoming available to arbitrageurs (specialist investors who exploit minute market inefficiencies). Indeed there exist mathematical formulae such as the Black Scholes option pricing formula which are regularly used to discover under or overvalued options (unlike equities, options can be sold short). So my final piece of evidence that equities are probably much riskier over the long term than is commonly believed is indeed the very fact that we find the cost of a put option at 6% to be surprisingly high!

(6.7) To be strictly accurate, this is the cost in the market place of an American put option, i.e. one which can be exercised early, whereas here we are really interested in the less valuable European put option. Using the Black Scholes formula to estimate the value of an at the money European put on equities for a 20 year period, with equity volatility at 20% per annum and an assumed cash real rate of 2% per annum leads to a premium of about 15% of the value of the equity portfolio. This again provides confirmation of the very significant risk of losing money in real terms over 20 years from a 100% equity portfolio.

(7) CONCLUSION

(7.1) I have presented some historical and statistical evidence that the risks of equity investment remain significant even for long-term investors. These risks can be mitigated via traditional common sense diversification (the adoption of a balanced portfolio spread across all major asset classes), or alternatively via the use of put options. Investors should be aware of the limitations of portfolio insurance.

(7.2) A distinction must be made between stable and unstable investment strategies. An investment strategy is unstable if it is almost certain to lead to losses if a majority of other investors adopt a similar strategy. Examples of unstable strategies are portfolio insurance and rigid tracking of the CAPS median. An example of a stable strategy (i.e. one which is not unstable) is constant mix (where the portfolio is rebalanced at regular intervals to restore the distribution of assets to predetermined proportions in the main asset classes - e.g. 70% equities, 30% bonds).

(7.3) The main object of this paper is to stimulate debate and further research in these areas. I look forward to examining the implications of more complicated investment models, including perhaps those arising from Chaos Theory!
REFERENCES


For a description of the actuarial techniques for determining a suitable long-term asset allocation for an institutional investor see the following (and the papers referenced therein):


(9.1) **A: The Independence Model for Investment Returns**

(9.2) Let the continuously compounded annual real rate of return $L(1)$ on a portfolio (in the paper I have looked at a portfolio of UK and international equities but in principle this could be any portfolio of securities - the parameters $\mu$ and $\sigma$ would depend on the securities involved) be normally distributed with mean and variance $\mu$ and $\sigma^2$.

We could express this as:

$$L(1) \sim N(\mu, \sigma^2)$$

(9.3) (Recall that $L(1)$ corresponds to $\delta$, the force of real interest on the portfolio for an annual period).

If an investor invests £1 in the portfolio at time 0, then the value $V(1)$ of the investor’s portfolio at the end of a year will be $\exp(L(1))$, which will be distributed lognormally, with the same underlying parameters $\mu$ and $\sigma^2$, since $L(1)$ is distributed normally with parameters $\mu$ and $\sigma^2$.

We could express this as:

$$V(1) \sim \log N(\mu, \sigma^2)$$

(9.4) The continuously compounded rate of return $L(t)$ on the underlying asset portfolio over not an annual period, but say a period of $t$ years ($t>0$, with $t$ an integer) will be, under the independence model, a sum of $t$ independent identically distributed random normal variables, each with mean $\mu$ and variance $\sigma^2$. Thus $L(t)$ will itself be normally distributed, with mean $t\mu$ and variance $t\sigma^2$, and so:

$$L(t) \sim N(t\mu, t\sigma^2)$$

(9.5) What about the value $V(t)$ of the investor’s portfolio at the end of the $t$ year period?

$$V(t) = \exp(L(t))$$ hence

$$V(t) \sim \log N(t\mu, t\sigma^2)$$

In fact this is true even if $t$ is not an integer.

(9.6) Recall that if $X \sim \log N(\mu, \sigma^2)$ then $X$ has mean and variance according to the following expressions:

- mean $X = \exp(\mu + \frac{1}{2}\sigma^2)$
- variance $X = \exp(2\mu + 2\sigma^2) - \exp(2\mu + \sigma^2)$

Thus $V(t)$ has mean: $\exp(t\mu + \frac{1}{2}t\sigma^2)$ and variance: $\exp(2t\mu + 2t\sigma^2) - \exp(2t\mu + t\sigma^2)$

The presence of the $\sigma$ term in the expression for the mean of $V(t)$ explains the “volatility bias” effect that I displayed in graph 5.
In order to calculate the \( p \)-th percentile \( x(p) \) for \( V(t) \), we need only observe that:

\[
\text{if } \text{Probability} \ (V(t) < x(p)) = p \quad \text{then} \\
\text{Probability} \ (\log V(t) < \log x(p)) = p \\
\text{ie} \quad \text{Prob.} \quad (N(\mu, \sigma^2) < \log x(p)) = p \\
\text{Prob.} \quad (N(0,1) < (\log x(p) - \mu)/\sigma \sqrt{t}) = p
\]

The \( p \)-th percentile, \( y(p) \), of the unit normal distribution can be obtained from standard tables, hence we can solve for \( x(p) \) from the following equation:

\[
(x(p) - \mu)/\sigma \sqrt{t} = y(p) \\
x(p) = \exp(\sigma \sqrt{t} y(p) + \mu)
\]

Turning to the distribution of the average return \( R(t) \) achieved on the investor's portfolio over the \( t \) year period, this is:

\[
R(t) = V(t)^{(1/t)} \\
\quad = (\exp L(t))^{(1/t)} \\
\quad = \exp(\frac{L(t)}{t}) \\
\text{Now } L(t)/t \sim N(\mu, \sigma^2/t) \\
\text{so } R(t) \sim \log N(\mu, \sigma^2/t) \\
\text{hence } R(t) \text{ has mean } \exp(\mu + \frac{1}{2} \sigma^2/t) \\
\text{and variance } \exp(2\mu + 2\sigma^2/t) - \exp(2\mu) + \sigma^2/t
\]

Similarly the \( p \)-th percentiles, \( z(p) \), of \( R(t) \) are calculated from:

\[
\text{Prob}(R(t) < z(p)) = p \\
\text{ie} \quad \text{Prob}(\log R(t) < \log z(p)) = p \\
\text{Prob}(\log R(t) - \mu/\sigma \sqrt{t} < (\log z(p) - \mu)/\sigma \sqrt{t}) = p \\
\text{Prob}(N(0,1) < (\log z(p) - \mu)/\sigma \sqrt{t}) = p \\
\text{ie} \quad (\log z(p) - \mu)/\sigma \sqrt{t} = y(p), \text{ again where } y(p) \text{ is the } p \text{-th percentile of the} \\
\text{standard } N(0,1) \text{ distribution.} \\
\text{So } z(p) = \exp((y(p)^2 \sigma \sqrt{t} + \mu)
\]

Tables 1 and 2 show the values of the various statistics for \( V(t) \) and \( R(t) \) illustrated graphically in the paper, using values for \( \mu \) and \( \sigma \) of \( \log(1.06) \) and 0.20 respectively.

**B: Positive Autocorrelation Models for Investment Returns**

The simplest form of such models are of the form:

\[
l(t) = \mu + a(l(t-1) - \mu) + \sigma e(t) \quad (\text{NB } 1 > a > 0) \\
\text{where:} \\
l(t) = \text{continuously compounded annual rate of return between time } t-1 \text{ and } t \\
\mu = \text{the long-term mean of the } l(t) \text{ process} \\
\sigma = \text{a parameter for the standard deviation of annual continuously} \\
\text{compounded returns} \\
\text{and} \\
e(t) = \text{a unit normal random variable, the } e(t) \text{ being an independent series of such} \\
\text{random variables}
\]
(9.13) Then if we start \( l(0) \) at \( \mu \),

\[
\begin{align*}
l(1) &= \mu + \sigma e(1) \text{ ie } l(1) \sim N(\mu, \sigma^2) \\
l(2) &= \mu + \sigma (\mu e(1)) + e(1) \\
l(3) &= \mu + \sigma (\mu e(1) + \mu e(2) + e(3)) \\
l(t) &= \mu + \sigma (\mu e(1) + \mu a e(2) + \ldots + e(t))
\end{align*}
\]

thus \( l(t) \) has mean \( \mu \) (since all the \( e() \)s have mean 0) and variance:

\[
\begin{align*}
\sigma^2(a^{2(0-1)} + a^{2(0-2)} + \ldots + 1) \\
= \sigma^2(1-a^2)/(1-a^2)
\end{align*}
\]

(the sum of a geometric series)

(We see here why \( a \) must be less than 1: if not, then the variance of \( l(t) \) tends to infinity and the process is unstable)

(9.14) Hence \( l(t) \), being a linear combination of the normal random variables \( e() \), is itself normally distributed:

\[ l(t) \sim N(\mu, \sigma^2(1-a^2)/(1-a^2)) \]

(9.15) What of the continously compounded return \( L(t) \) over the \( t \) year period? Unlike in the Independence Model, \( L(t) \) is no longer a sum of \( t \) independent normal random variables. Instead:

\[ L(t) = \sum_{j=1}^{t} l(j), \text{ and the } l(j) \text{ are not independent, but positively correlated.} \]

In fact, we can express \( L(t) \) solely in terms of the \( e() \)s:

\[
\begin{align*}
L(t) &= t\mu + \sigma(1+a+a^2+\ldots+a^{t-1}) e(1) + \\
&\quad (1+a+\ldots+a^{t-1}) e(2) + \ldots + (1)e(t) \\
&= t\mu + (\sigma/(1-a)) \sum (1-a^t) e(1) + \\
&\quad (1-a^t)e(2) + \ldots (1-a^t) e(t)
\end{align*}
\]

(9.16) Thus \( L(t) \), being a linear combination of normal random variables, is itself normally distributed, with mean \( t\mu \) (since all the \( e() \)s have mean 0) and variance (since the \( e() \)s are independent):

\[
\begin{align*}
\text{Var } L(t) &= (\sigma/(1-a)) \sum (1-a^t) + (1-a^t) + \ldots (1-a^t) \\
&= (\sigma/(1-a)) \sum (t - 2(a+a^2+\ldots+a^t)+(a^2+a^3+a^4+\ldots+a^t)) \\
&= (\sigma/(1-a)) \sum (t-2a(1-a^t)/(1-a)+a^2(1-a^t)/(1-a^2))
\end{align*}
\]

Now as \( t \) gets larger,

\[ \text{Var } L(t) \text{ tends to } (\sigma/(1-a))^2 \cdot (t) \]
(9.17) In other words by comparison with the Independence Model where the variance of \( L(t) \) was \( t \sigma^2 \), we note that the variance of \( L(t) \) has been increased by a factor \( 1/(1-a)^2 \). So when calculating percentiles for \( V(t) \) and \( R(t) \), the calculations will be as before, but replacing \( \sigma \) by \( \sigma/(1-a) \). For a value of \( a \) of 0.6, this is equivalent to increasing \( \sigma \) from 20% to 50%, a tremendous increase. Even if \( a = 0.2 \) (which might be thought too weak by those who favour this type of model), \( \sigma \) increases from 20% to 25%, which would still increase the risk of the portfolio significantly.

(9.18) What if more complicated models are used, with the return this year being dependent not just on last year’s return but on prior years? The calculations are more complex, but the effect will be the same: the variance of \( L(t) \) will increase relative to the Independence model, simply because as in the above example, when \( L(t) \) is expressed in terms of the \( e() \)s, further positive terms involving \( a \)'s will be added to the equation, whereas in the Independence case, \( L(t) \) is simply:

\[
L(t) = t \mu + \sigma e(1) + e(2) + ... + e(t)
\]

(9.19) Hence the conclusion that positive autocorrelation makes the probability of loss on a portfolio increase.

(9.20) **C: Negative Autocorrelation Models for Investment Returns**

(9.21) The simplest such model would be similar to the model in (B), but with a negative sign for the \( a \) term:

\[
L(t) = \mu - a( L(t-1) - \mu ) + \sigma e(t) \quad (\text{where } 0 < a < 1)
\]

In this case, the analysis proceeds exactly as for model (B), with \(-a\) substituted for \( a \). Thus as \( t \) gets large, the variance of \( L(t) \) tends to:

\[
t \sigma^2 / (1-(-a))^2 = t \sigma^2 / (1+a)^2
\]

ie with \( a = 0.6 \), \( \sigma \) would decrease to 12.5% from 20% and the smallest value of \( \sigma \) could be obtained by having \( a \) infinitesimally close to 1 (but not equal to 1, otherwise the process again becomes unstable), when \( \sigma \) is halved to 10%.
Table 1: Statistics for Returns (in %)

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As Graph 2 will show, this misleadingly underestimates the risk of poor equity performance.

Note the diminishing funnel of double 5th, Median, Mean and 95th Percentiles

Graph 1: Real Returns on Equities
Graph 2: Real Value of Equity Portfolio

5th, 50th, Mean and 95th Percentiles

Inflation Adjusted Value of Portfolio

Time in years

Note the expanding funnel of doubt! Portfolio values are what matters, not returns and this shows that equities remain risky over long periods.
**Graph 3: Real Value of Equity Portfolio**

5th Percentile versus Median Value of Portfolio

*Note: *- - median  - - - - 5th percentile

Inflation Adjusted Value of Portfolio

Time in years

- 5th percentile < 1 for many years
Note: ooo = mean 000 = median

Graph 4: Real Value of Equity Portfolio
Mean versus Median Value of Portfolio
Thus the higher the volatility, the higher the mean. This has implications for long-term investment.

Note: The two assets have the same (6%) but the asset represented by the - - - has a 20% asset represented by the - - - - -.

Two Assets: Identical except for Volatility

Graph 5: Volatility Bias Effect